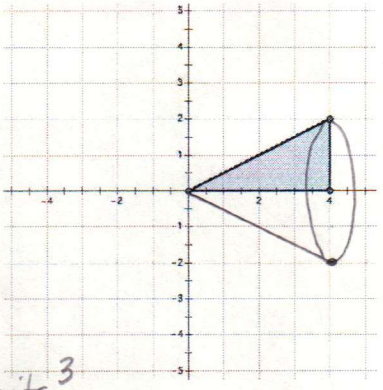
5.2 Any Way You Spin It

A Develop Understanding Task

Perhaps you have used a pottery wheel or a wood lathe. (A lathe is a machine that is used to shape a piece of wood by rotating it rapidly on its axis while a fixed tool is pressed against it. Table legs and wooden pedestals are carved on a wood lathe). You might have played with a spinning top or watched a figure skater spin so rapidly she looked like a solid blur. The clay bowl, the table leg, the rotating top and the spinning skater—each of these can be modeled as **solids of revolution**—*a three dimensional object formed by spinning a two dimensional figure about an axis.*

Suppose the right triangle shown below is rotating rapidly about the *x*-axis. Like the spinning skater, a solid image would be formed by the blur of the rotating triangle.



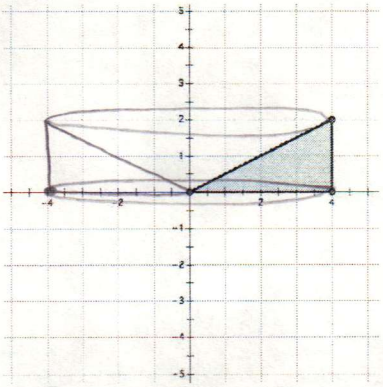
1. Draw and describe the solid of revolution formed by rotating this triangle about the *x*-axis.

   It makes a cone

2. Find the volume of the solid formed. $r = 2$ $h = 4$

   $V = \frac{1}{3}\pi r^2 h$

   $V = \frac{1}{3}\pi(4)(4) = 5\frac{1}{3}\pi$ or 16.76 units^3



3. What would this figure look like if the triangle rotates rapidly about the *y*-axis? Draw and describe the solid of revolution formed by rotating this triangle about the *y*-axis. It would make a cylinder that is missing a cone shape

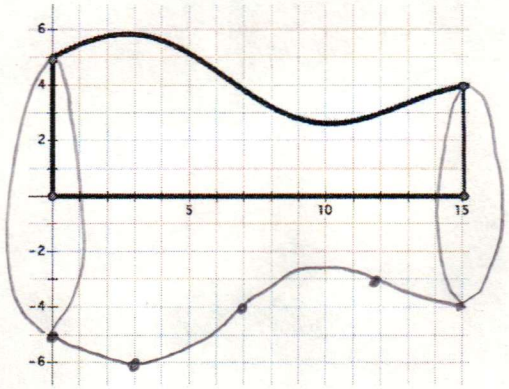
4. Find the volume of the solid formed. $r = 4$ $h = 2$

   $V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

   $V = \pi(16)(2) - \frac{1}{3}\pi(16)(2)$

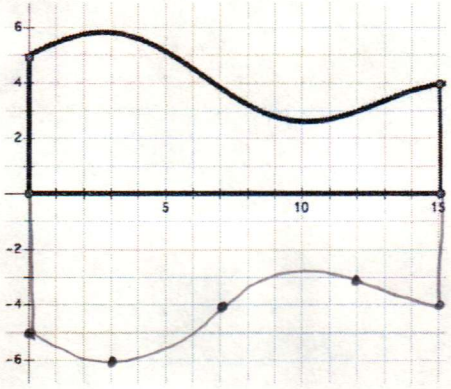
   $V = 32\pi - 10\frac{2}{3}\pi = 21\frac{1}{3}\pi$ or 67.02 units^3

5. What about the following two-dimensional figure? Draw and describe the solid of revolution formed by rotating this figure about the x-axis.



Looks like a vase

6. Draw a cross section of the solid of revolution formed by this figure if the plane cutting the solid is the plane containing the coordinate axes.



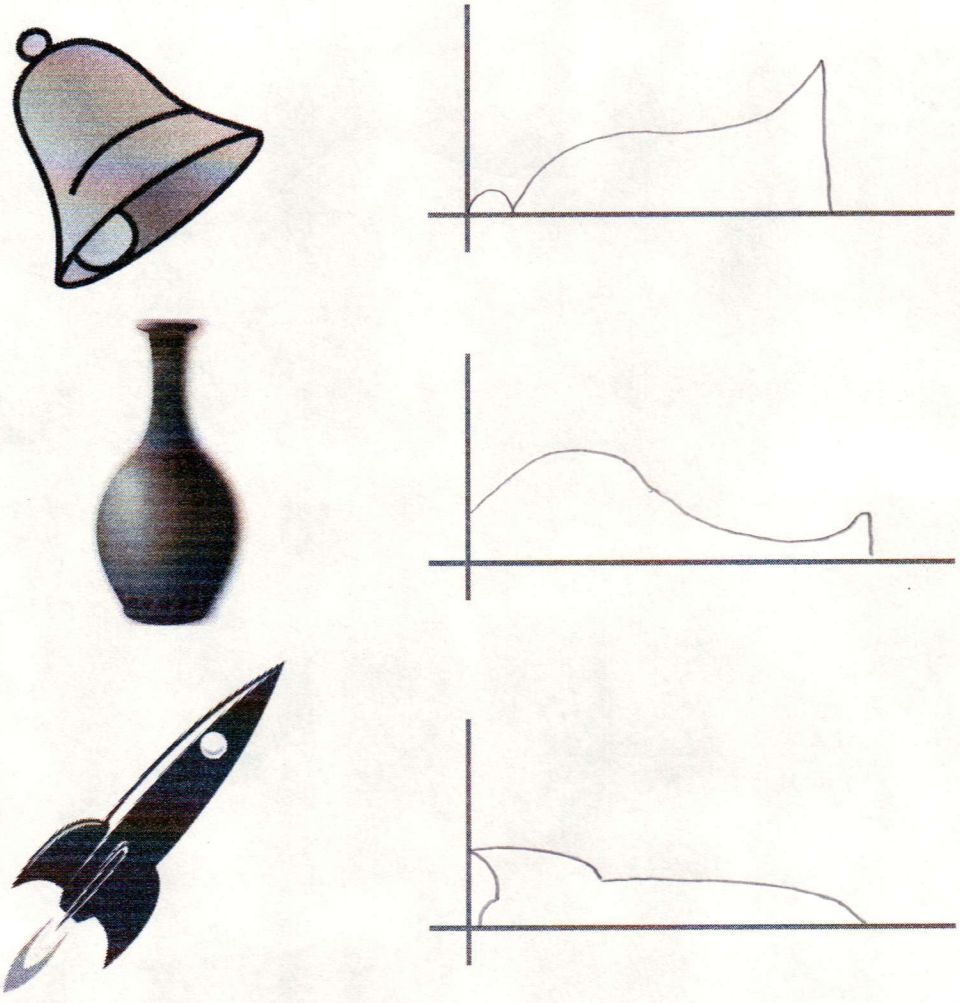
7. Draw some cross sections of the solid of revolution formed by the figure above if the planes cutting the solid are perpendicular to the plane containing the coordinate axes. Draw the cross sections when the intersecting planes are located at $x = 5$, $x = 10$ and $x = 15$.

3 different sizes of circles

So, why are we interested in solids that don't really exist—after all, they are nothing more than a blur that forms an image of a solid in our imagination. Solids of revolution are used to create

mathematical models of real solids by describing the solid in terms of the two-dimensional shape that generates it.

8. For each of the following solids, draw the two-dimensional shape that would be revolved about the *x*-axis to generate it.



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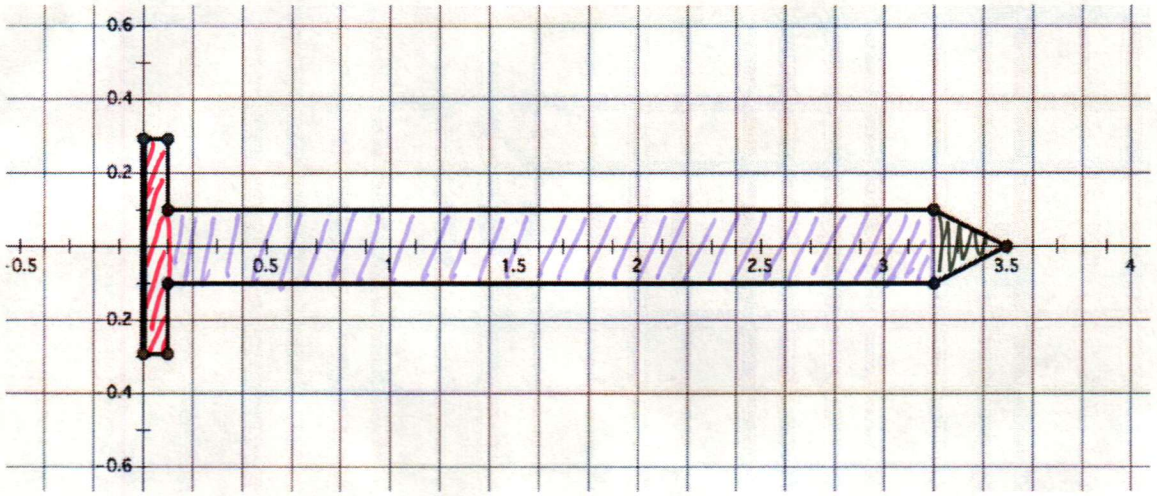
Volume Formulas	Area Formulas
Prism or Cylinder: $V = Base \bullet Height$	Rectangle: $A = LW$
Cone or Pyramid: $V = \frac{1}{3} Base \bullet Height$	Triangle: $A = \frac{1}{2} Base \bullet Height$
Sphere: $V = \frac{4}{3}\pi r^3$	Circle: $A = \pi r^2$



Cylinder $r = .3$ $h = .1$

$V = \pi(.3)^2 \cdot .1 = .028$

$V = .128 \text{ units}^3$

Cylinder $r = .1$ $h = 3.1$

$V = \pi(.1)^2 \cdot (3.1) = .097$

Cone $r = .1$ $h = .3$

$V = \frac{1}{3}\pi(.1)^2 \cdot (.3) = .003$